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REMARKS ON THE PARITY DETERMINATION OF NARROW RESONANCES*

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Recently several proposals were put forward to determine the parity of narrow baryonic resonances, in particular the Θ^+ . In these proceedings we will briefly comment on the general problems in this task and then discuss in detail the potential of reactions of the type $\vec{N}\vec{N} \rightarrow \Theta^+ Y$, where Y denotes a hyperon (either Σ or Λ) and the arrows indicate that a polarized initial state is required. Besides reiterating the model-independent properties of this class of reactions we discuss the physics content of some model calculations.

1. Generalities

The parity of a hadron contains significant information on its substructure. Unfortunately, especially for spin-1/2 particles, the determination of the parity is a non-trivial problem especially when we talk about narrow states.

To illustrate the origin of the difficulty we observe that—in leading order of the outgoing cms momentum—the decay vertex of a spin-1/2 resonance into a spin-1/2 particle and a pseudoscalar (e.g. $\Theta^+ \rightarrow NK$, if the Θ^+ were indeed a spin-1/2 particle as suggested by almost all models) reads $\vec{\sigma} \cdot \vec{q}$ for a positive parity resonance and 1 for a negative parity resonance. Thus, as long as we do not measure the polarization of the decay products, all observables scale as $(\vec{\sigma} \cdot \vec{q})(\vec{\sigma} \cdot \vec{q}) = \vec{q} \cdot \vec{q}$ or 1, accordingly, leaving no unique trace of the intrinsic parity of the decaying object.

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Unfortunately a measurement of the polarization of a nucleon in the final state is technically very demanding and of low efficiency. In addition, as was clearly demonstrated by Titov in this conference¹, possible interference phenomena with the background amplitudes put into question, if such a measurement would indeed allow to determine the parity of the resonance unambiguously.

Thus, the only straightforward option that remains is to extract the quantum numbers of this narrow resonance from a partial wave analysis. However, if the resonance of interest is very narrow also this program might be unsuccessful: in this case the partial wave analysis allows solely to put an upper limit on the width of the decaying particle^{5,4}—note, also the more sophisticated work of Ref.⁶, where data on K^+d scattering is used directly—here the KN partial wave analysis was used to fix the parameters of the model for the background amplitudes.

Does this mean that there is no way to determine the parity of a narrow resonance? The answer to this question is no, for one can use the stringent selection rules that are enforced by the Pauli Principle on the nucleon–nucleon (NN) systems to manipulate the total parity of a system. It is well known that a two–nucleon state acquires a phase $(-)^{L+S+T}$ under permutation of the two particles, where L , S , and T denote the angular momentum, the total spin and the total isospin of the two nucleon system. The required antisymmetry of the NN wavefunction thus calls for $L+S+T$ to be odd. For example, for a proton–proton state $T = 1$ and the parity is given by $(-)^L$ —thus each $S = 1$ state has odd parity and each $S = 0$ state has even parity. Therefore, preparing a pure spin state of a pp system means preparing a NN state of known parity. In case of a $T = 0$ state, the assignment of spin and parity needs to be reversed.

A well known textbook example that exploits this method is the measurement of the parity of the pion² in π^- capture on deuterium from an atomic s -state ($\pi^-d \rightarrow nn$); this transition would be forbidden if the parity of the spin 0 pion were positive. To see this we recall that an even parity nn state, characterized by an even value of L , is to be a spin singlet; consequently, in this case we have for the total angular momentum $J = L$. On the other hand, since for the deuteron $j = 1$ the initial state has $J = 1$. Since the parity of the whole system agrees to that of the pion, an s -wave production is allowed only for negative parity pions.

It was essential in case of the reaction $d\pi^- \rightarrow nn$ that the deuteron is a $j = 1$ state and the pion is a scalar—this fixed the total angular momentum to $J = 1$ for the s -wave initial state. For the system $NN \rightarrow Y\Theta^+$ the final

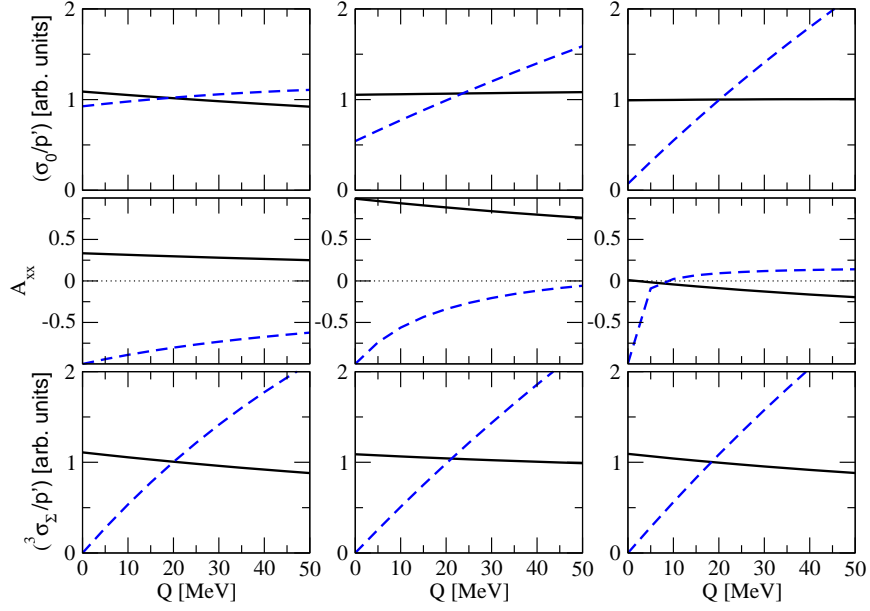


Figure 1. Energy dependence of the total cross section and the angular integrated polarization observable A_{xx} and ${}^3\sigma_{\Sigma}$ for the reaction $pp \rightarrow \Sigma^+\Theta^+$. Solid (dashed) lines correspond to a negative (positive) parity Θ^+ . Shown are results for three different models for the production operator: the left column shows the results for the model with only kaon exchange, the middle one those for the one with K^* exchange and the right one those for the model including K^* and K exchange. All results for σ_0 and ${}^3\sigma_{\Sigma}$ are normalized to 1 at an excess energy of 20 MeV and are divided by the phase-space volume.

state can be both spin singlet as well as spin triplet. Therefore in this case we need to manipulate the spin of the initial state to fix its parity. This observation was first made in Ref.⁷ and then further exploited in a series of publications^{3,8,9,10,11,12,13}. In addition, we will have to use the energy dependence of the spin cross sections to identify the leading partial wave, as was observed in Refs.^{7,13}.

2. The ideal observable

In terms of the so-called Cartesian polarization observables, the spin-dependent cross section can be written as¹⁷

$$\begin{aligned} \sigma(\xi, \vec{P}_b, \vec{P}_t, \vec{P}_f) = \sigma_0(\xi) \left[1 + \sum_i ((P_b)_i A_{i0}(\xi) + (P_f)_i D_{0i}(\xi)) \right. \\ \left. + \sum_{ij} ((P_b)_i (P_t)_j A_{ij}(\xi) + (P_b)_i (P_f)_j D_{ij}(\xi)) \right. \\ \left. + \sum_{ijk} (P_b)_i (P_t)_j (P_f)_k A_{ijk}(\xi) \dots \right] . \end{aligned} \quad (1)$$

where $\sigma_0(\xi)$ is the unpolarized differential cross section, the labels i, j and k can be either x, y or z , and P_b, P_t and P_f denote the polarization vector of beam, target and one of the final state particles, respectively. All kinematic variables are collected in ξ .

In Refs.^{15,17,18} it was shown, that a measurement of the spin correlation parameters A_{xx}, A_{yy}, A_{zz} as well as the unpolarized cross section allows to project on the individual initial spin states. More precisely

$$\begin{aligned} {}^1\sigma_0 &= \sigma_0(1 - A_{xx} - A_{yy} - A_{zz}) , \\ {}^3\sigma_0 &= \sigma_0(1 + A_{xx} + A_{yy} - A_{zz}) , \\ {}^3\sigma_1 &= \sigma_0(1 + A_{zz}) , \end{aligned} \quad (2)$$

where the spin cross sections are labeled following the convention of Ref. ¹⁷ as ${}^{2S+1}\sigma_{M_S}$, with S the total spin of the initial state and M_S its projection; σ_0 denotes the unpolarized cross section. Unfortunately, longitudinal polarization (needed for A_{zz}) is not easy to prepare in a storage ring. However, the following linear combination projects on spin triplet initial states and no longitudinal polarization is needed⁸:

$${}^3\sigma_\Sigma = \frac{1}{2}({}^3\sigma_0 + {}^3\sigma_1) = \frac{1}{2}\sigma_0(2 + A_{xx} + A_{yy}) . \quad (3)$$

For $\vec{p}\vec{p} \rightarrow \Theta^+\Sigma^+$, only negative parity states contribute to ${}^3\sigma_\Sigma$; thus only in case of a negative parity Θ^+ s -waves are allowed in the final state. If we assume the Θ^+ to be an isoscalar, the reaction $\vec{p}\vec{n} \rightarrow \Theta^+\Lambda$ gives the same amount of information, however, positive and negative parity change their roles.

It is well known that for large momentum transfer reactions in the near threshold regime the energy dependence of a partial wave characterized by angular momentum l is given by $(p/\Lambda)^l$, where Λ denotes the intrinsic scale

of the production process—for reactions of the type $NN \rightarrow B_1B_2$ this is typically given by the momentum transfer

$$p \sim \sqrt{(M_{B_1} + M_{B_2} - 2M_N)M_N} .$$

For an extensive discussion of this type of reaction we refer to Ref.¹⁵. It is thus sufficient to measure the energy dependence of $^3\sigma_\Sigma$ for small excess energies to pin down the parity of the Θ^+ : in case of a positive parity in the pp channel it scales as phase-space times an odd polynomial in the excess energy $Q = \sqrt{s} - \sqrt{s_0}$, where s_0 denotes the threshold energy. On the other hand, if the resonance has negative parity the energy dependence should be that of phase-space times an even polynomial. It should be stressed that these considerations apply only for outgoing cms momenta significantly smaller than Λ —for larger energies no general statement on the energy dependence is possible in a model-independent way. This is quite obvious once it is recalled that there are rigorous energy-dependent bounds on the strength of the individual partial waves set by unitarity—thus there should not be an unlimited growth.

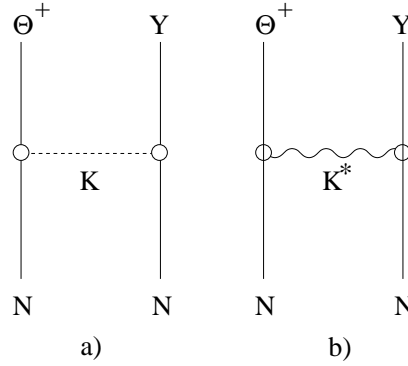
The near threshold properties of $^3\sigma_\Sigma$ are shown in the lower line of Fig. 1, where the results for different models (for more details about these see next section and the appendix of Ref.¹³) for the unpolarized cross section σ_0 , the spin correlation coefficient A_{xx} and $^3\sigma_\Sigma$ are shown. Although the energy dependence of σ_0 as well as A_{xx} is vastly different, reflecting the different admixture of partial waves in the different models, the figure clearly illustrates that the energy dependence of $^3\sigma_\Sigma$ is an unambiguous signal for the parity of the Θ^+ .

In Ref.¹³ we also investigate if the spin transfer coefficient D_{xx} can be used for a parity determination and we refer the interested reader to this paper. The remaining space available for these proceedings will now be used to discuss the reliability and features of model calculations for reactions of the type $NN \rightarrow B_1B_2$.

3. Remarks on models

As mentioned above the reactions $NN \rightarrow B_1B_2$ are characterized by a large momentum transfer. This has two consequences: first of all the energy dependence of the production process in the near threshold regime is fixed model-independently; secondly, it is very difficult to construct a reliable microscopic model for these reactions.

Probably the most clear illustration of the latter point is the fact that there is not even a microscopic model available to describe the data on

Figure 2. *Diagrams considered in the model calculations.*

$pp \rightarrow pp\pi^0$, although this is the first inelasticity of the NN system and much is known about all the subsystems. Only recently it was observed, that the large momentum transfer as it occurs in inelastic NN reactions leads to a relative enhancement of pion loop contributions (see discussion in Ref. ¹⁵). Given this it seems inappropriate to construct a model for the reaction $NN \rightarrow \Theta^+ Y$ that is quantitatively reliable, since here a lot less is known about possible production mechanisms—there might even be quite complicated production mechanisms of relevance, like the decay of a heavier resonance as proposed in Ref.¹⁶. In addition, it is well established that the initial state interaction can have a significant effect on observables. First of all it reduces the cross section typically by a factor of 2–3 and secondly it introduces an additional phase to the individual amplitudes; especially the latter effect can well change the shape especially of polarization observables, for they are quite sensitive to the relative phases of the contributing amplitudes. Note, in reactions of the type $NN \rightarrow \Theta^+ Y$, where the final state interaction is expected to be weak, the relative phase of the amplitudes is largely introduced by the NN interaction in the initial state. Through the Watson theorem¹⁹ this phase can be related to the NN scattering phase-shifts. Although this is true rigorously only in the elastic case, it is still reasonable to expect a properly adjusted formula to also work for the NN system at energies as high as relevant for the Θ^+ production²⁰. However, such a detailed work is beyond the scope of this paper—especially, the initial state interaction will not change significantly the energy dependence of observables, as long as only energies close to the production threshold are investigated.

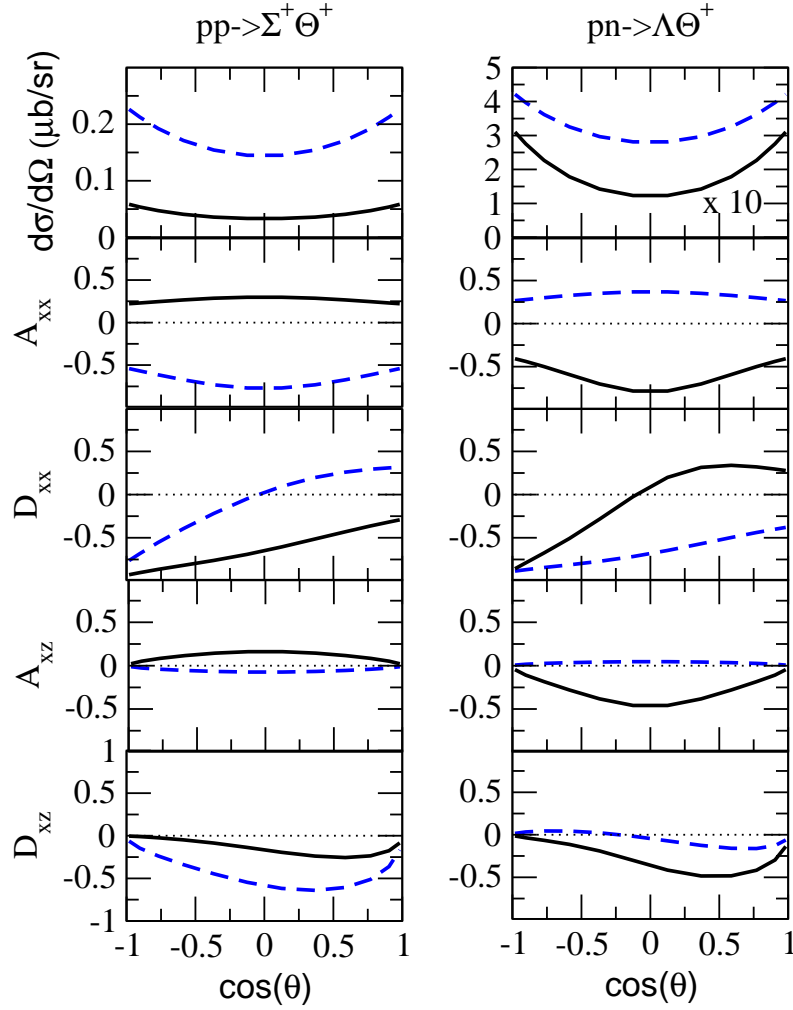


Figure 3. Angular distributions for the K -exchange model calculated at an excess energy of 40 MeV. The meaning of the curves is as in Fig. 1. The lower curve in the upper left panel is scaled by a factor of 10 (as indicated in the panel).

As a result of all this as in Ref.¹³ we here take a more pragmatic point of view: if all statements made above are indeed true model-independently, they have to apply to any (realistic) model. Since we do not trust the overall scale of the model results and for a better comparison of the results for the two different parities, the results for the integrated observables in

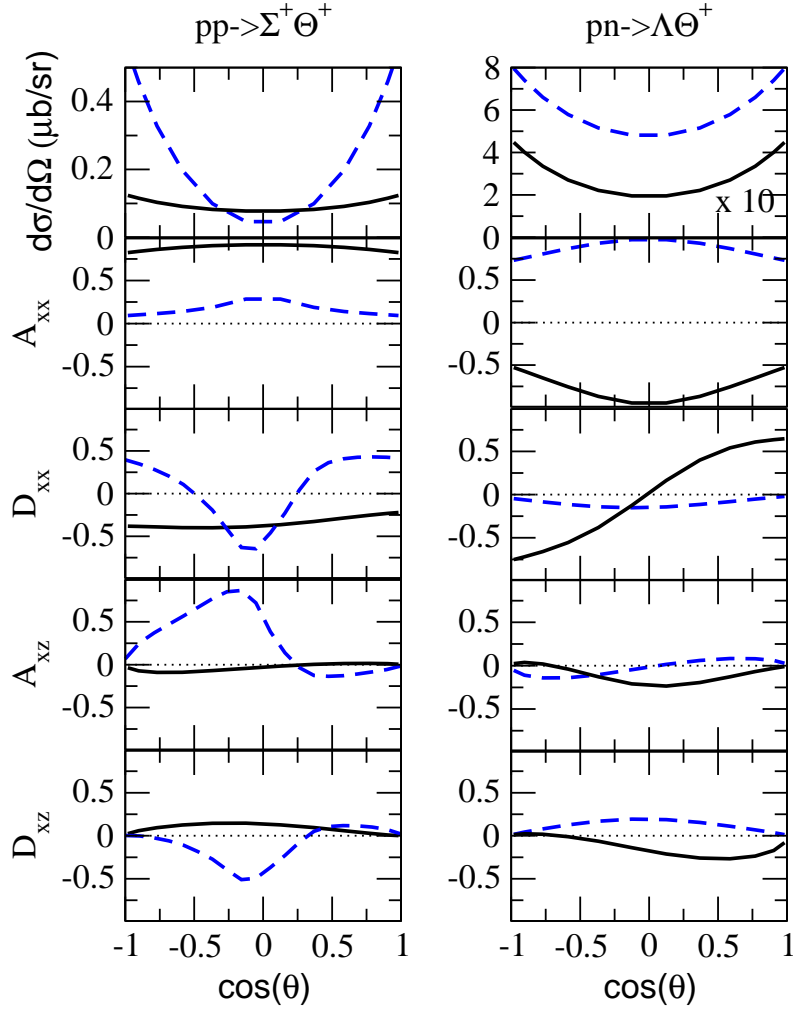


Figure 4. Angular distributions for the $K + K^*$ -exchange model calculated at an excess energy of 40 MeV. The meaning of the curves is as in Fig. 1. The lower curve in the upper left panel is scaled by a factor of 10 (as indicated in the panel).

Fig. 1 are normalized to 1 at 20 MeV.

We constructed a model where a scan through a wide range of parameters allowed to study a large class of different effects. To be more specific, we included the diagrams shown in Fig. 2, fixing the coupling strength of the K exchange and then varied the parameters for the K^* exchange. The

following three models turned out to be representative for the many cases we studied, namely a model with purely K exchange, a model with purely K^* exchange and a mixture of both. In the third case the relative strength of the two diagrams was adjusted such that for a positive parity Θ^+ the s -wave contributions of the two diagrams canceled in the pp reaction^a. This lead to a quite drastic energy dependence of A_{xx} .

It turned out that, although the energy dependence of the cross section as well as A_{xx} was very different for different models, the behavior of ${}^3\sigma_\Sigma$ was always as described above.

One question repeatedly asked on the conference was that about the possible production mechanisms. Given the problems mentioned above regarding the construction of the production operator for Θ^+ production in NN collisions, it will most probably not be possible to unambiguously identify a particular production mechanism as the most significant one from data on these reactions directly. However, what the polarization observables serve for is to exclude particular production mechanisms. To be more concrete: as different production mechanisms are typically characterized by different spin and isospin dependencies, they will lead to quite different polarization observables for the pp and pn induced reaction. This is illustrated in Figs. 3 and 4, where the angular distributions of various spin observables are shown for two different sets of model parameters (pure K exchange and $K + K^*$ exchange) for both channels. Clearly, the two models lead to very different angular dependencies of most of the observables shown. Thus, these data could well be used to exclude particular production mechanisms (once the initial state interaction is included as described above) and thus improve our understanding of the hadronic interactions of the Θ^+ (... if it exists).

4. Summary and outlook

In summary we have argued that the most promising method to model independently determine the parity of narrow resonances is a measurement of $\vec{N}\vec{N} \rightarrow B_1B_2$: the energy dependence of the spin triplet cross section, given by ${}^3\sigma_\Sigma = \frac{1}{2}\sigma_0(2 + A_{xx} + A_{yy})$, is the ideal observable. There is also a chance that $\sigma_0 D_{xx}$ also allows to determine the parity—for details on this we refer to Ref.¹³.

^aWe also unsuccessfully tried to construct a model were the same happens for the negative parity.

In addition, especially the angular dependence of the large number of existing polarization observables will put strong constraints on the allowed production mechanisms.

In this project theory did its part—now we have to wait for the experimental realization. We want to close with a few comments on the prospects of a measurement of A_{xx} for $NN \rightarrow \Theta^+ Y$. So far a measurement for the unpolarized cross section in the pp channel is completed by the TOF collaboration at the COSY accelerator²¹: with a statistical significance of about 4.5σ a total cross section of 400 nb was extracted from data on $pp \rightarrow \Sigma^+ p K^0$. At COSY polarized beams are routinely available and a frozen spin target is currently being adopted to the COSY conditions. The first double polarized measurement is expected for summer 2005.

Acknowledgments

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